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GOAL Problem # Go! Problem 1 of 11 Prev Next Leave

Question Difficulty: Core

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You fit the **sat1** data set to the following model:

$$Y_{tij} = \beta_0 + \beta_1 X_{tij} + u_i + v_j + \epsilon_{tij} \qquad \textbf{(Model 8.1)}$$

where $X_{tij} = \text{year}_{tij}$, $u_i \sim N(0, \sigma^2_{st})$, $v_j \sim N(0, \sigma^2_{te})$, and $\epsilon_{tij} \sim N(0, \sigma^2)$. Note that $X_{1ij} = -1$ indicates Grade 3 students, $X_{2ij} = 0$ indicates Grade 4 students, and $X_{3ij} = 1$ indicates Grade 5 students.

The summary output of fitting **Model 8.1** is:

```
> summary( PS8.1.fit )          # Model 8.1
Linear mixed model fit by REML ['lmerMod']
Formula: math ~ year + (1 | studid) + (1 | tchrid)
   Data: sat1

Random effects:
 Groups   Name        Variance Std.Dev.
 studid   (Intercept) 326.4    18.07
 tchrid   (Intercept) 843.8    29.05
 Residual             184.8    13.60
Number of obs: 229, groups:  studid, 122; tchrid, 12

Fixed effects:
            Estimate Std. Error t value
(Intercept)  596.987      8.747   68.25
year          28.891     10.057    2.87

Correlation of Fixed Effects:
     (Intr)
year 0.062
```

Calculate the marginal predicted value for Grade 5 students.

A 525

B 554

C 597

D 626

E 669

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The marginal predicted value is $\hat{Y}_{tij} = \hat{\beta}_0 + \hat{\beta}_1 \times X_{tij}$.

Solution

From the given output we have $\hat{\beta}_0 = 596.987$ and $\hat{\beta}_1 = 28.891$. For Grade 5 students, $X_{tij} = 1$.

Hence, the marginal predicted value is

$$\begin{aligned}\hat{Y}_{tij} &= \hat{\beta}_0 + \hat{\beta}_1 \times X_{tij} \\ &= 596.987 + 28.891 \\ &= 625.878\end{aligned}$$

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Question

Difficulty: Mastery

The Metropolis algorithm is chosen to sample the posterior distribution of a parameter θ.

Assume the following:

- An unnormalized posterior defined by the function
$$f(\theta) = \begin{cases} 0, & \text{for} \quad \theta < 0 \\ e^{-\theta/2}, & \text{for} \quad \theta \geq 0 \end{cases}$$
- A sampler initialization point of 3.0 before Iteration 1
- Each proposal in the Metropolis algorithm is the sum of the current position of the sampler and the step within a given iteration
- A proposal is accepted if the acceptance ratio, $f(\theta_{\text{prop}})/f(\theta_{\text{curr}})$, is greater than the random uniform number

The first four iterations for the sampler are provided in the table below.

Iteration	Step	Random Uniform Number
1	1.0	0.820
2	−1.4	0.939
3	2.2	0.233
4	−3.7	0.468

Calculate the position of the sampler after iteration 4.

A Less than 1.0

B At least 1.0, but less than 2.0

C At least 2.0, but less than 3.0

D At least 3.0, but less than 4.0

E At least 4.0

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Let s be the step (starting from Step=1.0), we have $\theta_{\text{P}} = \theta_{\text{C}} + s$. The acceptance ratio is

$$\frac{f(\theta_{\text{P}})}{f(\theta_{\text{C}})} = \frac{e^{-\theta_{\text{P}}/2}}{e^{-\theta_{\text{C}}/2}} = e^{-s/2}.$$

Solution

Let s be the step, we have $\theta_{\text{P}} = \theta_{\text{C}} + s$. The acceptance ratio is

$$\frac{f(\theta_{\text{P}})}{f(\theta_{\text{C}})} = \frac{e^{-\theta_{\text{P}}/2}}{e^{-\theta_{\text{C}}/2}} = e^{-s/2}.$$

For the first iteration, we have $\theta_0 = 3$ and $s_1 = 1$. Hence, $\theta_{\text{C}} = 3$ and $\theta_{\text{P}} = 3 + s_1 = 4$. The acceptance ratio is $e^{-s_1/2} = e^{-1/2} = 0.607$. The ratio is less than the first random number 0.82 and hence we should stay at the current point: $\theta_1 = \theta_{\text{C}} = 3$.

For the second iteration, we have $\theta_1 = 3$ and $s_2 = -1.4$. Hence, $\theta_{\text{C}} = 3$ and $\theta_{\text{P}} = 3 + (-1.4) = 1.6$. The acceptance ratio is $e^{-s_2/2} = e^{1.4/2} = 2.014$. The ratio is greater than one (and certainly greater then 0.939) and hence we should move to the proposed point: $\theta_2 = \theta_{\text{P}} = 1.6$.

For the third iteration, we have $\theta_2 = 1.6$ and $s_3 = 2.2$. Hence, $\theta_{\text{C}} = 1.6$ and $\theta_{\text{P}} = 1.6 + 2.2 = 3.8$. The acceptance ratio is $e^{-s_3/2} = e^{-2.2/2} = 0.333$. The ratio is greater than the third random number 0.233 and hence we should move to the proposed point: $\theta_3 = \theta_{\text{P}} = 3.8$.

For the fourth iteration, we have $\theta_3 = 3.8$ and $s_4 = -3.7$, we have $\theta_{\text{C}} = 3.8$ and $\theta_{\text{P}} = 3.8 + (-3.7) = 0.1$. The acceptance ratio is $e^{-s_4/2} = e^{3.7/2} = 6.36$. The ratio is greater than one and hence we should move to the proposed point: $\theta_4 = \theta_{\text{P}} = 0.1$.